REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. REPORT DATE *(DD-MM-YYYY)*
05-09-2003

2. REPORT DATE
FINAL REPORT

3. DATES COVERED *(From - To)*
15/03/1999 to 14/03/2002

4. TITLE AND SUBTITLE
Advanced Wavelet Methods for Image and Signal Processing

5a. CONTRACT NUMBER
N00014-99-1-0547

5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER

5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER

6. AUTHOR(S)
DeVore, Ronald A.

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
University of South Carolina
901 Sumter St. 5th Floor
Sponsored Programs & Research
Columbia, SC 29208

8. PERFORMING ORGANIZATION REPORT NUMBER

20031003 146

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

11. SPONSORING/MONITORING AGENCY REPORT NUMBER

12. DISTRIBUTION AVAILABILITY STATEMENT
Approved for public release; distribution unlimited.

13. SUPPLEMENTARY NOTES

14. ABSTRACT
The research on this contract was directed towards areas of mathematics and numerical computation which have applications to image/signal processing and PDEs. The research can be broadly classified into the following areas: (i) image processing, (ii) processing Digital Terrain Elevation Data (DTED), (iii) information theory, (iv) analogue to digital conversion, (v) approximation and (vi) theoretical and numerical PDEs.

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF: a. REPORT	b. ABSTRACT	c. THIS PAGE	17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
					19b. TELEPONE NUMBER (*Include area code*)

ADVANCED WAVELET METHODS FOR IMAGE AND SIGNAL PROCESSING

Principal Investigator: Ronald A. DeVore

FINAL TECHNICAL REPORT

Contract Number: N00014–99–1–0547

Overview: Research of the Principal Investigator on this contract was directed towards areas of mathematics and numerical computation which have applications to image/signal processing and PDEs. The research can be broadly classified into the following areas: (i) image processing, (ii) processing Digital Terrain Elevation Data (DTED), (iii) information theory, (iv) analogue to digital conversion, (v) approximation, and (vi) theoretical and numerical PDEs. We briefly summarize the highlights of our work in these areas.

Image Processing

We have concentrated on techniques which are important for automated target recognition. The first step is usually to compress and remove noise from the image. Later steps include feature extraction and image registration. Some highlights of our work are the following:

Image Compression.

We continue our efforts to understand image compression through mathematical analysis. The paper [CDDD] introduces a new concept of tree approximation and shows its fundamental role in the construction of modern image encoders. In particular, we use these ideas to create an encoder which is progressive, optimal (in the sense of Kolmogorov entropy) for classes of images modelled on Besov smoothness, and allows burnin. The burnin property, reported on in [DJPS] allows the user to specify a subregion of the image and subsequent bits in the bitstream will only pertain to this subregion. Further results on tree approximation, namely how to find optimal trees, is given in [BDKY]. A paper in preparation [BDP] shows how one can obtain near best tree approximants of an unknown function with $O(n^2)$ computations through querries. This result is important in PDE solvers. Our recent work is directed at video encoding including encoding in a wireless environment. We are developing wavelet flow algorithms to predict subsequent frames in video (from an initial frame) which is an attractive substitute for motion compensation and optic flow predictions.

Noise Suppression.

The space BV of functions of bounded variation is often used as a model for real images (an idea of Mumford and Shah). Therefore this space appears in important extremal problems resting at the core of noise reduction in images. In [CDPX], we make the **remarkable**

discovery that wavelet thresholding is a minimizer for the K–functional between L_2 and BV. Heretofore, this minimization problem was solved only using PDE techniques which are numerically intensive. The proof that wavelet techniques also minimize is deep and depends on a new adaptive algorithm for piecewise constant approximation. The numerical implementation of these results give visually comparable performance to PDE based algorithms with far less computation.

In [CDKP], we analyze thresholding algorithms used in statistical estimation and show that to each such algorithm one can associate a certain maximal space which benchmarks their performance. The main tool for obtaining maximal spaces is to develop an approximation theory [CDH] which describes error rates when the error is measured in L_p (not necessarily $p = 2$) while the thresholding is done relative to another L_τ norm.

Digital Terrain Elevation Data (DTED)

USC has assembled a team (led by the PI) of research faculty, postdoctoral, graduate and undergraduate students who are working to develop multiscale methods for the rapid processing of large data sets with a particular emphasis on the application of this processing platform to DTED maps and associated imagery. Several algorithms are currently under investigation for efficient and accurate processing of DTED maps for registration, denoising, compression, and rendering operations.

Some of the desired features of the algorithms under development are: (a) high compression, (b) robust error handling, (c) progressive transmission of the data, (d) quick rendering, and (e) burning in (tunneling), and (f) line of sight display. Since almost all graphic hardware uses triangular polygonal patches as building blocks for object description, we have focused our attention on algorithms utilizing meshes of polygonal elements. Two classes of algorithms are being investigated:

- Nonlinear approximation algorithms based on adaptive multiresolution analysis,
- Greedy (insertion or removal) algorithms for mesh construction which utilize Delaunay triangulations.

The first algorithms include: (a) initial course adaptive triangulation which allows a low resolution good approximation, (b) wavelet decomposition of the function for achieving sparse representation of the function (surface), (c) conversion to hierarchical B-spline representation and application of the nonlinear uniform approximation schemes adapted from our previous work on adaptive approximation, (d) compression and progressive transmission of the data using the hierarchical representation.

The greedy removal algorithm is a recursive procedure with the following basis elements: (a) determination and upgrading of the significance table of the grid points, (b) removal one by one of the least significant points, (c) mesh updating after each removal with Delaunay triangulation algorithm. The greedy insertion algorithm utilizes the same elements but in a reverse order. We pay special attention to the data structures that enable us to compress and transmit the data progressively.

Although we have developed methods for n-term piecewise polynomial approximation which provide the best *asymptotic* rates of approximation, practical implementation issues will ultimately determine the extent to which these methods are useful in applications. We have formed an algorithm implementation team to develop research software incorporating both the nonlinear approximation and greedy methods mentioned earlier. There is a common modularized framework (OPTA) for possible variations of the algorithms which will provide the flexibility of designing and testing proposed hybrid methods, and will allow us to compare our results with those of other researchers. The various components are built upon a common elemental base of hierarchical structures naturally suited for multiresolution analysis, data operations, and rendering of the surfaces. Specialized data structures have been tailored to take advantage of particular attributes of the approximation algorithms under investigation. In order to test the composite approximation and rendering schemes under a variety of computing environments, code development is in C, using the GLUT libraries for windowing and OpenGL language for graphics rendering. Special consideration has been given to insuring that rendering algorithms take full advantage of hardward available on a given platform. Additional attention in the development is being paid to future options of distributed and massively parallel development, since the algorithms are inherently scalable.

The potential applications of this technology include mission planning and rehearsal, autonomous navigation, rapid registration of imagery to DTED, post battle assessment, rapid telecommunication, and efficient touring, querying and storage of large data bases.

Analogue to Digital (A/D) Converters

There is great demand for efficient analogue to digital converters in both the civilian and military sectors. State of the art methods are the Sigma–Delta converters which consist of three steps: (i) oversampling the signal, (ii) one bit quantization of the oversampled values, and (iii) filter reconstruction. Sigma–Delta converters work well but there was no mathematical understanding of why. Daubechies and DeVore [DD] began to mathematically analyze sigma–delta converters and have been able to quantify their success. They have also derived higher order methods which theoretically perform better than existing converters. These methods are being analyzed further for their practicality and implementation in silicon.

There is presently a small consortium consisting of Daubechies, DeVore, and researchers at AT&T, Lucent, and Hewlett Packard, who are investigating the performance of the new sigma-delta architectures. One of the main goals of this collaborative effort is to explain the success of Sigma–Delta modulation. Toward that end, we have shown in [DDGV] that Sigma–Delta has the remarkable feature that it is self correcting for quantization error. This property is not inherited by Pulse Code Modulation (PCM) which is built on the Shannon sampling. Further investigations have been sparked by the fact that PCM has exponential decreasing distortion as a function of bit rate while Sigma–Delta methods only have polnomial decrease. We have found in [DDGV] that it is possible to use redundancy to create encoders which have exponential decreasing distortion rate while retaining the quantization error correction of Sigma–Delta, i.e. one has the best of both worlds.

Information Theory

In [DVDD], we have given a survey and state of the field accounting of the role of Harmonic Analysis in information theory, especially data compression. The main point of this article is that advances in information theory parallel advances in harmonic analysis. Also, harmonic analysis gives the theoretical underpinnings which explain the connections between the Shannon (stochastic) theory and the Kolmogorov (deterministic) theory.

We are currently developing an information theory description applicable in other settings such as DTED. The elements of this theory are models for DTED data sets, metrics to measure the distortion commensurate with applications (Hausdorff metric and line of sight metrics) and encoders which perform at optimal bit rate. As a sample theorem we mention that we have shown that the elements in the space BV of functions of bounded variation can be encoded in the Hausdorff metric to distortion $O(1/n)$ using n bits whereas this set is not even compact in the stronger L_∞ metric (and hence cannot be encoded at all in that metric).

Approximation Theory

We continue our work in approximation theory which we feel provides a theoretical underpinning for many application areas such as image processing and PDE solvers. We have introduced tree approximation in [CDDD] since it models both image encoders as well as adaptive PDE solvers. The work in [CDDD] provides the fundamental rate of approximation bounds for this form of nonlinear approximation. In [BDKY], we have characterized precisely the functions which have a prescribed rate of tree approximation through certain maximal operators. In [BDP], we give fast numerical methods for the implementation of tree approximation. It is shown that it is possible to find near best tree approximants for an unknown function f using only $O(n^2)$ querries of its wavelet coefficients. The importance of this paper lies in PDE solvers where the solution is not known and so it is not clear which wavelet coefficients to evaluate in an adaptive numerical scheme.

We have also done fundamental work on approximation using a library of bases. Here the problem is given a class F of target functions and a collection B of bases, to find the best basis for f to be used in conjunction with n-term approximation. The remarkable work of Donoho shows that in a Hilbert space setting with the library of orthonormal bases it is often possible to determine the best basis for the class F *a priori*. In [DPT], we generalize Donoho's work to more general L_p metrics and show that the competition for best bases can be enlarged to the class of all unconditional bases which are democratic.

Theoretical and Numerical PDEs

An important consequence of our work in [CDPX] is the proof that a function in BV has wavelet coefficients in weak ℓ_1. This result has had considerable resonance in PDEs. Yves

Meyer has shown that our results give new embedding inequalities (Poincaré, Gagliardo–Nirenberg) for Besov spaces which are important in PDEs. He then conjectured other improvements which could not be derived from the weak ℓ_1 theorems for the wavelet coefficients of BV functions.

In a series of papers (the first two are [CDD], [CDD1]), we are developing adaptive wavelet based algorithms for solving operators equations. These papers are the first to derive adaptive PDE solvers whose rates of convergence can be determined *a priori* – in fact, we produce an algorithm whose error bounds are shown to be optimal in terms of its rate of approximation of the unknown solution u to the operator equations.

RECENT RESEARCH PAPERS ORIGINATING FROM ONR FUNDING

[BDP] P. Binev, R. DeVore, and P. Petrushev, *Fast Algorithms for Adaptive Tree Approximation*, Numerische Mathematik, to appear.

[BDKY] R. Baraniuk, R. DeVore, G. Kyriazis, and X. M. Yu, *Best Tree Approximation*, Advances in Comp. Math **16** (2002), pp. 357–373.

[CDD] A. Cohen, R. DeVore, and W. Dahmen, *Adaptive Wavelet Methods for Elliptic Operator Equations: Convergence Rates*, Math. Comp. **70** (2000), pp. 27–75.

[CDD1] A. Cohen, R. DeVore, and W. Dahmen, *Adaptive Wavelet Methods for Operator Equations: Beyond the Elliptic Case*, J. FoCM **2**, No. 3 (2002), pp. 203–245.

[CDDD] A. Cohen, W. Dahmen, I. Daubechies, and R. DeVore, *Tree Approximation and Encoding*, ACHA **11** (2001), pp. 192–226.

[CDDD1] A. Cohen, W. Dahmen, I. Daubechies, and R. DeVore, *Harmonic Analysis of the Space BV*, Revista Matematica Iberoamericana **19** (2003), pp. 235–263.

[CDH] A. Cohen, R. DeVore, and R. Hochmuth, *Restricted Nonlinear Approximation*, Constructive Approximation **16**, No. 1 (2000), pp. 85–113.

[CDKP] A. Cohen, R. DeVore, G. Kerkyacharian, and D. Picard, *Maximal Spaces with Given Rate of Convergence for Thresholding Algorithms*, ACHA **11** (2001), pp. 167–191.

[CDPX] A. Cohen, R. DeVore, P. Petrushev, and H. Xu, *Nonlinear Approximation and the space $BV(R^2)$*, Amer. J. of Math **121** (1999), pp. 587–628.

[DD] I. Daubechies and R. DeVore, *Reconstructing a Bandlimited Function From Very Coarsely Quantized Data: A Family of Stable Sigma-Delta Modulators of Arbitrary Order*, Annals of Mathematics **158** (2003), pp. 643–674.

[DDGV] I. Daubechies, R. DeVore, and S. Güntürk, *Exponential Precision in A/D Conversion with an Imperfect Quantizer*, preprint.

[DJPS] R. DeVore, S. Johnson, C. Pan, and R. Sharpley, *Optimal Entropy Encoders for Mining Multiply Resolved Data*, Data Mining II, N. Ebecken and C. A. Brebbia (eds.), WIT Press, Boston, 2000, pp. 73–82.

[DPT] R. DeVore, G. Petrova, and V. Temlyakov, *Best Basis Selection for Approximation in L_p*, J. of FoCM **3** (2003), pp. 161–185.

[DVDD] D. Donoho, M. Vetterli, R. DeVore, and I. Daubechies, *Data Compression and Harmonic Analysis*, IEEE Trans. Information Theory **44** (1998), 2435–2476.